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Reference Systems and Gravitation

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Abstract

It is shown how the formalism of the tetrad theory of gravitation used by Treder (1967a, b, 1970) follows from the more general fibre bundle formalism. This is of interest in the study of the relations between tetrad theories and the general theory of relativity. In particular, the breaking of the principle of general relativity and the interpretation of tetrad fields as reference systems are considered in greater detail.

1. *Introduction*

The tetrad theory of gravitation proposed by Treder (1967a, b) uses differential equations for quantities $h^A{}_i$ which are vectors with respect to coordinate transformations in the space-time V_4, and which can also be considered as vectors with respect to local Lorentz rotations in the tangent spaces $\tau_x(V_4)$ of V_4. It is useful to see how this formalism developed by Treder (1967a, b, 1970) follows from the more general fibre bundle formalism by making largely physical assumptions (compare Section 2). Firstly, this enables us to use theorems of the theory of fibre bundles, a fact that is especially interesting for global investigations which so far have not been done. Secondly, it proves to be useful for the consideration of the principle of general relativity. We consider this latter aspect for tensorial and spinorial fields in connection with the problem of physical reference systems in Section 3 and 4.

2. *Fibre Bundle Formalism and Tetrad Theory of Gravitation*

In order to study the tetrad theory of gravitation, we consider the principal fibre bundle $\mathscr{P}$ of reference systems of the Riemann space-time V_4.† This bundle $\mathscr{P}$ may be defined by the equivalence class of the following

† See for instance: Steenrod (1951) and Auslander & Mackenzie (1963).

principal coordinate bundles:

$$\mathscr{P} = (P(V_4), V_4, Gl(4), (U_\alpha, \phi_{\alpha\beta}))$$

where V_4 is the base manifold, $P(V_4)$ is the union of all P_x (P_x denotes the set of all reference systems at the point x, i.e. the set of all ordered bases $\mathbf{X}_1, \ldots, \mathbf{X}_4$ of the tangent space $\tau_x(V_4)$), $Gl(4)$ is the general four-dimensional linear group, $\{U_\alpha\}$ is any covering of V_4 by coordinate neighbourhoods,† and $\phi_{\alpha\beta}$ is the transformation of the fibre $Gl(4)$ induced by the coordinate transformation $\bar{x}^i = \bar{x}^i(x^k)$ on $U_\alpha \cap U_\beta$; $\phi_{\alpha\beta}$ is an element of $Gl(4)$.

An element of the equivalence class of principal coordinate bundles representing the principal fibre bundle $\mathscr{P}$ may be constructed by choosing a basis $\mathbf{X}_1, \ldots, \mathbf{X}_4$ at each point x of a coordinate neighbourhood U_α. In this way we obtain a representation of each vector $\mathbf{Y}_i$ of any reference system by matrices $h^A{}_i$:

$$\mathbf{Y}_i = h^A{}_i \mathbf{X}_A \tag{2.1a}$$

This relation defines the map

$$\pi^{-1}(U_\alpha) \rightarrow U_\alpha \times Gl(4) \tag{2.1b}$$

because $h^A{}_i \in Gl(4)$. (π is the projection of $P(V_4)$ onto the base manifold V_4.)

The transformation $\phi_{\alpha\beta}$ of the fibre $Gl(4)$, that is the transformation of the matrices $h^A{}_i$ associated with the coordinate transformation $\bar{x}^i = \bar{x}^i(x^k)$ on $U_\alpha \cap U_\beta$, depends on the selection of the principal coordinate bundle; that is, it depends on the choice of the vectors $\mathbf{X}_A$. Suppose we choose the basis

$$\mathbf{X}_A \equiv \mathbf{X}_a = \frac{\partial}{\partial x^a} \tag{2.2}$$

Then

$$\bar{h}^k{}_i = \frac{\partial \bar{x}^r}{\partial x^l} h^l{}_i \tag{2.3}$$

In this case, the group of the transformations $\phi_{\alpha\beta}$ is reduced to the group of holonomic transformations.

From the physical standpoint of the tetrad theory of gravitation it now follows that we have to consider principal fibre bundles whose group can be reduced to the identity: It is the central idea of the tetrad theory of gravitation to regard a tetrad field $\mathbf{Y}_1, \ldots, \mathbf{Y}_4$ as the mathematical description of the gravitation, where the inner product of the vectors $\mathbf{Y}_i$ (on the tangent space $\tau_x(V_4)$)

$$g_{ir} = \langle \mathbf{Y}_i, \mathbf{Y}_k \rangle = \eta_{AB} h^A{}_i h^B{}_k \tag{2.4}$$

$$\mathbf{Y}_i = h^A{}_i \mathbf{X}_A \tag{2.5}$$

† In the tetrad theory of gravitation we are dealing with manifolds covered by only one coordinate neighbourhood. But, for reasons of a convenient description of coordinate transformations in fibre bundle theory we write $\{V_\alpha\}$.

provides the metric g_{ik} of the space-time V_4. We see from this that the small Latin indices, running from 1 to 4, become tensor indices with respect to coordinate transformations $\bar{x}^i = \bar{x}^i(x^k)$; but, in general, they are numbers under any transformations in the tangent spaces $\tau_x(V_4)$.† Compared to it, the Latin capitals of $h^A{}_i$ are tensor indices under any linear base transformations in the tangent spaces $\tau_x(V_4)$; its behaviour under coordinate transformations $\bar{x}^i = \bar{x}^i(x^k)$ on $U_\alpha \cap U_\beta \subset V_4$ given by $\phi_{\alpha\beta}$ depends on the choice of the principal coordinate bundle representing the principal fibre bundle. If there is a principal coordinate bundle belonging to the equivalence class of the principal fibre bundle such that its group is the identity, then the Latin capitals are not indices, but numbers under coordinate transformations. (In this case one can say that the group of the bundle can be reduced to the identity.)

Since the gravitational field is to be described by four linearly independent vector fields, one has, from the first, to consider manifolds V_4 which allow the global existence of such vector fields. Now, it is known from the differential geometry that, in the case of a differentiable n-dimensional base manifold, the group of its principal fibre bundle can be reduced to the identity if and only if the base manifold has n linearly-independent vector fields.‡ That is, the group of $\mathscr{P}$ considered in the tetrad theory of gravitation is reducible to the identity. Therefore, there are principal coordinate bundles such that the capital indices of $h^A{}_i$ are numbers with respect to coordinate transformations in V_4.

Now, in tetrad theory of gravitation one does not consider the complete principal fibre bundle, but only that subclass of principal coordinate bundles whose group is equal to the identity (in particular, one does not consider principal coordinate bundles with (2.2) and (2.3)).

It is clear from relation (2.4) that the elements of the subclass of principal coordinate bundles considered arise from each other by x-dependent Lorentz transformations of the bases $\mathbf{X}_1, \ldots, \mathbf{X}_4$ (the bases fix the single principal coordinate bundles):

$$\mathbf{X}_A' = \omega_A{}^B(x^i)\,\mathbf{X}_B, \qquad \omega_A{}^C(x^i)\,\omega^B{}_C(x^i) = \delta_A{}^B \tag{2.6}$$

This statement is another formulation of the principle of general relativity given by Treder (1970) and Treder & Liebscher (1970).

In this way the mathematical formalism used by Treder (1967a, b, 1970) follows from the more general fibre bundle formalism. The concept developed by Treder (1967a, b, 1970) starts by defining matrices $h^A{}_i$, where the capital indices are tensor indices under Lorentz transformations and numbers in the Riemann space-time V_4, and where the small indices are numbers under Lorentz transformations and tensor indices in V_4,

† We write 'in general' because, naturally, coordinate transformations can also be interpreted in the tangent spaces τ_x of V_4.

‡ See for example, Auslander & MacKenzie (1963), p. 179.

i.e. with respect to coordinate transformations in V_4. These $h^A{}_i$ give the metric

$$g_{ir} = \eta_{AB} h^A{}_i h^B{}_k$$

3. *The Principle of General Relativity in Tetrad Theories*

In the tetrad theory of gravitation the gravitational field is described by four vectors $\mathbf{Y}_i$ which obey the relation by Treder & Liebscher (1970). Thus one has to formulate field equations determining the vectors $\mathbf{Y}_i$. If one formulates equations for the components $h^A{}_i$ which represent the vectors $\mathbf{Y}_i$ with respect to the basis $\mathbf{X}_A$, then in order to determine the $\mathbf{Y}_i$, one has also to fix the basis $\mathbf{X}_A$. According to the principle of general relativity, this basis is only fixed up to local Lorentz transformations.†

The determination of the basis $\mathbf{X}_A$ leads to a violation of the principle of general relativity and enables a physical interpretation of the quantities $h^A{}_i$ fixed by the field equations. In the fibre bundle language, the violation of the principle of general relativity means the selection of a special principal coordinate bundle.

This necessary determination of the basis $\mathbf{X}_A$ follows automatically from the physical starting-point of the tetrad theory of gravitation developed by Einstein and Abraham in 1912 (Treder, 1971). According to this point of view, the nature of gravitation insists on the necessary transition from inertial to non-inertial reference systems, where, in general, the non-inertial reference systems cannot be embedded in a flat space-time. Therefore it follows from

$$\mathbf{Y}_i = h^A{}_i \mathbf{X}_A = \delta^B{}_i \Omega_B{}^A \mathbf{X}_A \tag{3.1}$$

that the vectors $\mathbf{X}_A$ are identical with the inertial reference systems in Minkowski space and the field equations determine the non-holonomic and non-Lorentz transformations $h^A{}_i$ and $\Omega^A{}_B$, respectively, transforming the inertial reference systems into the non-inertial reference systems which are associated with a gravitational field. For vanishing gravitation the following relation then holds:

$$\mathbf{Y} = \delta^A{}_i \mathbf{X}_A \tag{3.2}$$

Therefore the description of gravitational fields by four-vectors $\mathbf{Y}_i = h^A{}_i \mathbf{X}_A$ leads to a violation of the principle of general relativity, because in order to formulate field equations for the quantities $h^A{}_i$, the determination of the basis $\mathbf{X}_A$ is necessary; in other words, one has to select a principal coordinate bundle. In the tetrad theory of gravitation this problem is solved by the 'initial-value condition' for vanishing gravitational

† The choice of the coordinate system defines the basis corresponding to the small indices, but not the basis corresponding to the capital indices. Naturally, it is not necessary to determine the basis $\mathbf{X}_A$ or the corresponding basis in the two-dimensional complex spinor spaces, as long as only tensorial quantities are considered (compare Section 4).

fields. From this condition it follows that the basis $\mathbf{X}_A$ is identical with the reference systems of the special theory of relativity.

4. *Reference Systems and Measurable Values*

In Section 2 and 3 we preferred the viewpoint that the matrices $h^A{}_i(x^l)$ are components of four-vector fields $\mathbf{Y}_i(x^r)$ with respect to any, in general, non-integrable vector fields $\mathbf{X}_A(x^r)$ on V_4. That is, we considered them as vectors with respect to local Lorentz rotations. But because the capital indices are numbers in V_4, the functions $h^A{}_i(x^r)$ are also components of four-vector fields with respect to the bases $\mathbf{e}^i$ in V_4, which are reciprocal to $\partial/\partial x^i$. That is, any field equations for $h^A{}_i$ determine four-vector fields in V_4. Thus, for tensorial quantities we need not refer to any bases $\mathbf{X}_A$ in the tangent spaces $\tau_x(V_4)$. In the following this is discussed. We shall also show how these vector fields $h^A{}_i$ can be interpreted as reference systems for tensorial quantities.

Let V_4 be covered by a coordinate system $\{x^l\}$, and let $h^{*A}{}_i$ be four linearly independent vector fields. After Schmidt's method one can construct four vector fields $h^A{}_i$ from these $h^{*A}{}_i$ such that

$$h^A{}_i\, h^B{}_r\, \eta_{AB} = h^A{}_i\, h_{Ar} = g_{ir} \tag{4.1a}$$

and

$$h^i{}_A\, h^k{}_B\, g_{ir} = h^i{}_A\, h_{iB} = \eta_{AB} \tag{4.1b}$$

(The Minkowski metric η_{AB} is the metric of the tangent space τ_x at each point x). Now, it is known that also each linear combination $\bar{h}^A{}_i$

$$\bar{h}^A{}_i = \omega^A{}_B\, h^B{}_i \tag{4.2}$$

is pseudo-orthonormal if $\omega^A{}_B(x^l)$ obeys the relation

$$\omega^A{}_B\, \omega_C{}^B = \omega_B{}^A\, \omega^B{}_C = \delta^A{}_C \tag{4.3a}$$

where

$$\omega^A{}_B = \eta^{AC}\, \eta_{BD}\, \omega_C{}^D \tag{4.3b}$$

Indeed, using (4.2) and (4.3a, b) one gets

$$\bar{h}^A{}_i\, \bar{h}^B{}_k\, \eta_{AB} = \eta_{AB}\, \omega^A{}_C\, \omega^B{}_D\, h^C{}_i\, h^D{}_k = g_{ir} \tag{4.4a}$$

and

$$\bar{h}^i{}_A\, \bar{h}^r{}_B\, g_{ir} = \omega_A{}^C\, \omega_B{}^D\, h^i{}_C\, h^r{}_D\, g_{ir} = \eta_{AB} \tag{4.4b}$$

According to Section 2, the transformation (4.2) of $h^A{}_i$ associated with the transformation (2.6) of the basis $\mathbf{X}_A$ can be considered as Lorentz rotation with the coefficients $\omega^A{}_B$ in the tangent spaces $\tau_x(V_4)$. For tensor fields and its equations of motion (field equations), however, this dual formulation is formal. Here purely space-time operations are sufficient. In particular, the operation (4.2) can be considered as construction of four new space-time vectors without reference to any abstract rotations in the flat tangent spaces $\tau_x(V_4)$. But this does not hold for genuine spinor fields, since these entities are just defined as representations of the Lorentz rotations in τ_x, more exactly, as representations of the corresponding unimodular transformations.

The fact that purely space-time constructions are not sufficient to define and to interpret spinors, indicates that spinors are not measurable quantities. Therefore, referring only to directly measurable, i.e. tensorial quantities and, accordingly, considering the $h^A{}_i$ as space-time vectors, the formulation of field equations for the $h^A{}_i$ means a breaking of the principle of general relativity.

A tetrad $h^A{}_i$ may be considered as mathematical description of three standard rods and one standard clock. A measurement of a vector A^i, for instance, consists then in a projection of A^i upon the tetrad system $h^A{}_i$ giving the scalars $h^B{}_i A^i$; these scalars are the measurable values of A^i. Four arbitrary space-time scalars A^B represent the measurable values $h^B{}_i A^i$ of a space-time vector if and only if they are defined so that the choice of a new linear combination (4.2) as reference system is associated with the transition A^B into $\bar{A}^B$, where

$$\bar{A}^C = \omega^C{}_B A^B \tag{4.5}$$

That is, four scalars A^B represent the measurable values $h^B{}_i A^i$ of a space-time vector A^i if and only if they are Lorentz vectors with respect to the Lorentz transformations (4.2).

In accordance with the expectation noted above, that genuine spinors are no measurable quantities, we find that an analogous interpretation for spinors is impossible. The spinors are, from the first, scalars with respect to coordinate transformations in V_4. But these scalars are not measurable values, because they cannot be considered as the result of a measurement, i.e. as a projection upon measuring scales.

For as long as we consider only a flat Minkowski manifold E_4 and choose pseudo-Cartesian coordinates $\{x^l\}$ the metric has the form

$$g_{ir} = \eta_{ik} \tag{4.6}$$

Then the general orthonormal Schmidt tetrad takes the form

$$h^A{}_i = \omega^A{}_B \delta^B{}_i \tag{4.7a}$$

Using the matrix $\omega^{-1A}{}_B = \omega_B{}^A$, one can construct the linear combinations

$$\tilde{h}^A{}_i = \delta^A{}_i = \omega_B{}^A \omega^B{}_C \delta^C{}_i \tag{4.7b}$$

the $\tilde{h}^A{}_i$ are the inertial reference systems. In general coordinates $\bar{x}^r = \bar{x}^r(x^l)$ the tetrads (4.7a) and (4.7b) have the form

$$h'^A{}_i = \varphi^k{}_{,i}\, \omega^A{}_B \delta^B{}_r = \omega^A{}_B \varphi^B{}_{,r} \tag{4.8a}$$

and

$$\tilde{h}'^A{}_i = \varphi^A{}_{,i} \tag{4.8b}$$

In Riemann manifolds V_4 global inertial reference systems do not exist. Therefore, the construction of the Schmidt tetrad (4.7a) is possible only in an infinitesimal region (the construction of a local geodetic reference system, for instance, at $x = 0$).

Let us consider now the reference system that describes the proper rest system of a laboratory L, and let us compare it with reference systems fixed by any field equations.

Let $\mathscr{L}$: $x^i = x^i(s)$ be the world-line of L and $\hat{h}^A{}_i$ the reference tetrads representing the proper rest system of L along the world-line $\mathscr{L}$:

$$\hat{h}^A{}_i = \{u_i, N^1_i, N^2_i, N^3_i\} \tag{4.9}$$

where $u^i = dx^i/ds$ is the tangent vector to $\mathscr{L}$ and N^1_i, N^2_i, N^3_i are the three normal vectors. The proper rest systems $\hat{h}^A{}_i$ at the points $x_{\mathscr{L}}$ of $\mathscr{L}$ are carried into each other by the Fermi-Walker displacement:

$$\hat{h}^A{}_{i;l}\, u^l - \hat{h}^A{}_r(u_i\, u^r{}_{;l}\, u^l - u^k\, u_{i;l}\, u^l) = 0 \tag{4.10}$$

where $u^i{}_{;l} u^l = bN_1{}^i$. This displacement leaves the projection $\hat{A}^B = \hat{h}^B{}_i A^i$ of any vector A^i upon the tetrad $\hat{h}^A{}_i$ invariant, i.e., if both the tetrads $\hat{h}^A{}_i$ and all vectors (and also tensors of higher rank) are carried by this Fermi-Walker displacement along $\mathscr{L}$, then the measurable values $\hat{A}^B = \hat{h}^B{}_i A^i$ remain unchanged. Thus this Fermi-Walker displacement along $\mathscr{L}$ is equal to the Einstein teleparallelism using the $\hat{h}^A{}_i$ as reference tetrads. We have the relations

$$\begin{aligned} \hat{h}^i{}_B \hat{A}^B{}_{,l}\, u^l &= (A^i{}_{,l} + A^r \hat{h}^i{}_A \hat{h}^A{}_{r,l})\, u^l \\ &= (A^i{}_{;l} + A^r \hat{h}^i{}_A \hat{h}^A{}_{r;l})\, u^l \\ &= [A^i{}_{;l} - A_r(u^i u^r{}_{;l} - u^k u^i{}_{;l})]\, u^l = 0 \end{aligned} \tag{4.11}$$

where $\hat{h}^i{}_A \hat{h}^A{}_{k,l}$ is the Einstein connexion $\Delta^i{}_{rl}$.

From the relation (4.10) it is clear that if the curve is a geodesic, the Fermi-Walker displacement (4.10) goes over into the free Levi-Civita displacement. Then the motion of the laboratory L is a free one, and the $\hat{h}^A{}_i$ are a generalisation of the inertial tetrads $\tilde{h}^A{}_i$ in the sense of special relativity. We have then

$$\tilde{h}^A{}_{i;l}\, u^l = 0, \qquad u^i{}_{;l}\, u^l = 0 \tag{4.12}$$

Hence we get, instead of (4.11),

$$\begin{aligned} \tilde{h}^i{}_B \hat{A}^B{}_{,l}\, u^l &= (A^i{}_{,l} + A^r \tilde{h}^i{}_A \tilde{h}^A{}_{r,l}) \\ &= A^i{}_{;l}\, u^l = 0 \end{aligned} \tag{4.13}$$

Therefore, with regard to the tetrad system $\hat{h}^A{}_i = \tilde{h}^A{}_i$, the free Levi-Civita displacement and Einstein's teleparallelism are the same along $\mathscr{L}$.

If we introduce Fermi coordinates along $\mathscr{L}$ such that we have

$$(g_{ir})_{\mathscr{L}} = 0, \qquad (g_{ir,l})_{\mathscr{L}} = 0 \tag{4.14}$$

along $\mathscr{L}$, then we get for the orthonormal tetrad field along $\mathscr{L}$

$$(h^A{}_i)_{\mathscr{L}} = \omega^A{}_B \delta^B{}_i \tag{4.15a}$$

The linear combination

$$\delta^A{}_i = \omega_B{}^A \omega^B{}_C \delta^C{}_i \tag{4.15b}$$

gives then the tetrads $\tilde{h}^A{}_i$.

In general, the adaption of the tetrads $h^A{}_i$ to the proper rest systems of any laboratory L is excluded when, additional to the field equations following from (4.1a, b), equations for the $h^A{}_i$ are formulated. In particular, we find this situation when the sixteen $h^A{}_i$ are determined by means of sixteen differential equations (plus initial- and boundary-conditions). Then the physically privileged reference tetrads $\mathsf{h}^A{}_i(x^l)$ are solutions of these sixteen differential equations and the metric is given by the orthonormalisation condition (4.1a, b):

$$g_{ir} = \eta_{AB} \mathsf{h}^A{}_i \mathsf{h}^B{}_r \tag{4.16}$$

Using these vector fields $\mathsf{h}^A{}_i$, we can define the measurable values

$$\mathsf{A}^B = \mathsf{h}^B{}_i A^i \text{ etc.} \tag{4.17a}$$

on the whole manifold V_4. The values (4.17a) remain unchanged when the vector A^i is carried by the Einstein displacement

$$A^i{}_{,l} = -\Delta^i{}_{rl} A^r = -\mathsf{h}^i{}_A \mathsf{h}^A{}_{r,l} A^r \tag{4.17b}$$

In general, the $\mathsf{h}^A{}_i$ are different from the $\hat{h}^A{}_i$ along $\mathscr{L}$, because the displacement (4.17b), in general, is not Fermi-Walker displacement along $\mathscr{L}$. We have

$$\mathsf{h}^A{}_i = \omega^A{}_B \hat{h}^A{}_i \tag{4.18a}$$

In particular, if $\mathscr{L}$ is a geodesic we get, along $\mathscr{L}$,

$$\mathsf{A}^B{}_{,l} u^l = \mathsf{h}^B{}_{i;l} A^i u^l = \omega^B{}_{C,l} \tilde{A}^C u^l \tag{4.18b}$$

Starting from the fact that only tensorial quantities are measurable, the determination of the privileged reference tetrads $\mathsf{h}^A{}_i$ follows from the simultaneous satisfaction of the equations describing the interaction between gravitation and matter:

$$16 \text{ field equations for the } \mathsf{h}^A{}_i \text{ containing } \mathsf{T}_A{}^B \tag{4.19a}$$

$$\mathsf{h}^A{}_i \mathsf{h}^B{}_k \eta_{AB} = g_{ir} \tag{4.19b}$$

$$T_i{}^r{}_{;k} = 0 \tag{4.19c}$$

That is, for the reference tetrads $\mathsf{h}^A{}_i$ we have to demand: the measurable values

$$\mathsf{T}_A{}^B = \mathsf{h}_A{}^i \mathsf{h}^B{}_k T_i{}^k \tag{4.20}$$

of the matter tensor $T_i{}^k$ have to determine, by means of (4.19a) and (4.19b), that metric g_{ik} that appears in (4.19c).

For instance, let us consider a physical system which is described by the matter tensor T_i^k and let us assume that this system is both transmitter and receiver of gravitational radiation. If no other gravitational fields exist, then (4.19a) determines the gravitational waves, and (4.19c) describes the interaction between gravitational waves and the system under consideration. Then the reference tetrads are given by those $\mathbf{h}^A{}_i$, for which the measurable values (4.20) determine, by means of (4.19a) and (4.19b), that tensor g_{ik} that gives the reaction (4.19c) of the gravitational radiation on the physical system.

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